Lesson 19
Product Rules for Exponents

Outline

Part A
Product and Power Rules for Exponents

- Rule 1: Product of Powers
- Rules 2 and 3: Power Rules for Exponents

Part B
Applications of Exponent Rules 1–3

- More Than One Exponent Rule
- Formulas with Exponents

Targeted Review

Vocabulary

- power

Part A: Product and Power Rules for Exponents

Objectives

In this part of the lesson, you will learn about product and power rules for exponents.

By the end of this lesson, you will be able to do the following:

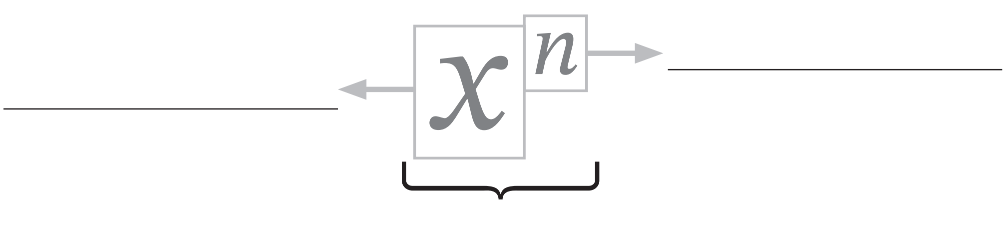
- Use the product of powers rule for exponents $\left(a^x \cdot a^y = a^{x+y}\right)$ to simplify expressions.
- Use the power of powers rule for exponents $\left(\left(a^x\right)^y = a^{x \cdot y}\right)$ and the power of a product rule for exponents $\left((ab)^x = a^x b^x\right)$ to simplify expressions.

Why?

The product and power rules for exponents allow you to multiply and factor terms that are raised to a power. You will use these rules throughout this unit and in future lessons.

Warm Up

Label the diagram using exponent, base, power, and term.



Rule 1: Product of Powers

- Adding like terms changes the ____________________, but *multiplying* like terms changes the ____:
    - $x + x =$ ____________________
    - $x \cdot x =$ ____________________
- The ____________________ (or exponent) represents the number of times that you multiply the base by itself.
- When the power is not visible, its value is ____________________.

- The rules for exponents are used when you are ____________________ bases together.
- Rule 1 is the product of powers rule for exponents:
  - When ____________________ bases are multiplied together, the exponents of those bases are ____________________.
  - Rule 1 product of powers: For all real numbers, ______________________________.

Example 1

Expand the exponential expression. Then simplify the expression to one term using the product of powers rule.

A) $x^5 \cdot x^3$

Expanded:

Simplified using the product of powers rule: $x^5 \cdot x^3 = x^{(5+3)} = x^8$

B) $a^4b^2 \cdot a^3b$

Expanded:

Simplified using the product of powers rule:

$a^4b^2 \cdot a^3b$

Example 2

Expand the exponential expression. Then simplify the expression to one term using the product of powers rule.

A) $5^4 \cdot 5^2$

Expanded: $(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5)$

Exponent Rule: $5^4 \cdot 5^2 = 5^{4+2} = 5^6$

B) $2^3 3^4 \cdot 2^2 3^2$

Expanded:

Use the Commutative Property to reorder the terms:

Exponent Rule:

> Our focus is on the exponents, but if you want to know this numerical value, you first need to find the value of each base.
> $2^5 = 32$
> $3^6 = 729$
> $32 \cdot 729 = 23{,}328$

Example 3

Find the missing number using the product of powers rule.

Solve for the value of the missing exponent. The simplified answer is given, but the exponent of one of the bases is missing. As long as *all of the bases are the same*, you can remove the bases and compare the exponents.

A) Find the value of the missing exponent.

$$x^3 \cdot x^? = x^{11}$$
$$3 + ? = 11$$
$$? = 11 - 3$$
$$? = 8$$

B) Find the value of n.

$$2^n \cdot 2^6 \cdot 2^7 = 2^8$$

The exponent rules work for all real number exponents, including exponents that are negative numbers or fractions.

Checkpoint

Expand the expression. Then simplify using the product of powers rule for exponents.

$$2^3a^2 \cdot 2^8a^5$$

Expanded:

Exponent Rule:

Rules 2 and 3: Power Rules for Exponents

- Rule 2 is the power of a ____________________ rule for exponents:
  - When a power is raised to a power, the exponents are ____________________.
  - For all real numbers, ________________________.
- Rule 3 is the power of a ____________________ rule for exponents:
  - If more than one base within parentheses is raised to a power, the exponent is ________________________ to *each* base.
  - For all real numbers, ________________________.
- Most often, a ________________________ of Rules 2 and 3 is needed.
  - For all real numbers, ________________________________.

Example 4

Expand the exponential expression using the outermost exponent. Then simplify the expression using the power of a power rule.

A) $(x^{30})^2$

Expanded: $x^{30} \cdot x^{30} = x^{30+30} = x^{60}$

Exponent rule: $x^{30 \cdot 2} = x^{60}$

B) $\left(x^{\frac{3}{2}}\right)^3$

Expanded:

Exponent rule:

Example 5

Simplify the expression using exponent rules. Write any numerical coefficients without an exponent.

A) $(a^4b^7c)^3$

$a^{4 \cdot 3}b^{7 \cdot 3}c^{1 \cdot 3}$

$a^{12}b^{21}c^3$

B) $(2x^3y^4)^4$

☑ Checkpoint

Explain the difference between Rule 1 and Rule 2.

Simplify.

A) $x^3 \cdot x^3$

B) $(x^3)^3$

Practice 1

Complete practice problems on a separate sheet of paper.

For problems 1–2, expand.

1) x^5

2) 6^2x^4

For problems 3–15, simplify. Assume all variables are positive.

3) $x^8 \cdot x^2$

4) $2^8 \cdot 2^{11}$

5) $3^5 \cdot 3^8 \cdot 3^{-3}$

6) $x^2 \cdot x^2 \cdot x$

7) $x^2y^2 \cdot xy^5$

8) $x^3y^9 \cdot x^2y$

9) $x^8y^{-2}z^{-3} \cdot xy^4z^8$

10) $2^2y^3 \cdot 2^3y^8$

11) $5^8xy \cdot 5x^2y^4$

12) $2^95^3 \cdot 2^65^{11}$

13) $(ab)(ab)$

14) $(3^7x^5) \cdot (5^5x^{12})$

15) $(a^2b^2c^2)(abc)$

For problems 16–18, find the value of n.

16) $y^5 \cdot y^3 \cdot y^n = y^{15}$

17) $x^{-3} \cdot x^n \cdot x^5 = x^8$

18) $(11^n)^7 = 11^{35}$

For problems 19–30, simplify.

19) $(17^8)^{\frac{1}{2}}$

20) $\left(x^{\frac{2}{3}}\right)^{18}$

21) $\left(y^{\frac{11}{3}}\right)^{\frac{3}{2}}$

22) $(a^3)^3$

23) $(x^9)^5$

24) $(y^3)^2$

25) $(x^2y^3)^5$

26) $(xy^2)^3$

27) $(p^8q^{14})^{\frac{1}{2}}$

28) $(x^2y)^4$

29) $(ab^3c)^7$

30) $(p^{27}q^{39})^{\frac{1}{3}}$

For problems 31–33, simplify. Write any numerical coefficients without an exponent.

31) $(2x)^2$

32) $(5y)^3$

33) $(3x^8)^3$

Mastery Check

Show What You Know

Complete the statement with one of the following words: always, sometimes, never.

If you use the word sometimes or never, provide a counterexample that shows why it is not always true.

A) The exponent rules are ____________________ true for all real numbers.

B) When using the product and power rules for exponents (Rules 1, 2, and 3), you will

____________________ *add* the exponents.

C) Simplify the expression using exponent rules. Write any numerical coefficients without an exponent.

$x^2 \cdot \left(3y^5\right)^2$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Practice 2

Complete practice problems on a separate sheet of paper.

For problems 1–2, simplify. Assume all variables are positive.

1) $x^{15} \cdot x^{-12} \cdot x$

2) $a^{\frac{2}{3}} \cdot a^{\frac{2}{3}} \cdot a^{\frac{5}{3}}$

For problems 3–4, simplify. Write your final answer with no exponents.

3) $2^3 \cdot 3^2$

4) $5^2 \cdot 2^4$

For problems 5–8, simplify.

5) $a^5b^2 \cdot a^7b^8$

6) $9^9x^2y^3 \cdot (9^3x^2y)$

7) $(m^3n^7) \cdot (m^{14}n^{-5})$

8) $(xyz) \cdot (x^2yz^3)$

For problems 9–10, simplify. Assume all variables are positive.

9) $3x^2 \cdot 2x^3$

10) $8xy^2 \cdot 3xy^2$

For problems 11–13, simplify.

11) $(12^5)^3$

12) $(26^2)^4$

13) $(5^7)^8$

For problems 14–15, find the value of *n*.

14) $5^n \cdot 5^{-6} \cdot 5^3 = 5^1$

15) $(a^3)^n = a^{33}$

For problems 16–18, simplify. Write improper fractions where needed.

16) $(2^8)^{\frac{2}{3}}$

17) $\left(x^{\frac{1}{2}}\right)^{12}$

18) $\left(8^{\frac{1}{2}}\right)^2$

For problems 19–27, simplify.

19) $(12^5y)^2$

20) $(x^5y^8)^4$

21) $(7^4x)^5$

22) $(a^2b^3c^4)^2$

23) $(m^2n^8)^9$

24) $(4xy^2)^4$

25) $(a^{11}b^5)^8$

26) $(m^{20}n^6)^{\frac{1}{2}}$

27) $(x^9b^{16})^2$

For problems 28–30, simplify. Write the numbers as a single term.

28) $(2^2 \cdot 3)^2$

29) $(5x^2yz)^3$

30) $(8^2x^8y^{20})^{\frac{1}{2}}$

Part B: Applications of Exponent Rules 1–3

Objectives

In this part of the lesson, you will learn about applications of exponent rules 1–3.

By the end of this lesson, you will be able to do the following:

- Simplify exponential expressions using both the product and power rules for exponents.
- Apply the product and power rules for exponents to formulas.

Why?

How can you determine the optimal size for a container? Being able to apply exponent rules to formulas will allow you to answer real-life questions like this.

Warm Up

Use the formula to find the area or volume of the figure.

1) Find the area of a rectangle with a length of 12 yards and a width of 5 yards.

2) Find the area of a square with sides of 9 feet.

3) Find the volume of a cube, $V = s^3$, when each edge has a length of 2 inches.

More Than One Exponent Rule

- Use the ______________________________ to determine which parts of an expression to simplify first.
- An expression is considered ______________ when each base occurs only ______________.

Example 1

Simplify.

$\left(2x^8y^6\right)^3 \cdot \frac{3}{5}x^{-3}$

Plan Determine the exponent rules needed to simplify.

Implement	Explain
$\left(2x^8y^6\right)^3 \cdot \frac{3}{5}x^{-3}$	◀ Given
$2^{1\cdot 3}x^{8\cdot 3}y^{6\cdot 3} \cdot \frac{3}{5} \cdot x^{-3}$	◀ Rules 2 and 3
$2^3 \cdot \frac{3}{5} \cdot x^{24+(-3)}y^{18}$	◀ Commutative Property
$8 \cdot \frac{3}{5} = \frac{24}{5}$	◀ Simplify numerical bases
$\frac{24}{5}x^{21}y^{18}$	◀ Rule 1

Example 2

Simplify.

$(8m^4n^3)^7 \cdot (3^4m)^5$

Plan Determine the exponent rules needed to simplify.

Implement	Explain
$(8m^4n^3)^7 \cdot (3^4m)^5$	◀ Given
	◀ Rules 2 and 3
	◀ Commutative Property
	◀ Rule 1

☑ Checkpoint

Simplify.

$(5a^4b^3)^8 \cdot (5a^6)^2$

Formulas with Exponents

- When working with formulas and terms that contain exponents, start with the

  ________________________________ and then substitute in the known values.

Example 3

Solve for the area of a square with sides of $11p^4q$ units.

Plan	Implement	Explain
$A = s^2$ $s = 11p^4q$ Use substitution and solve for A.	$A = \left(11p^4q\right)^2$	◀ Substitution
	$A = 11^{1\cdot 2}p^{4\cdot 2}q^{1\cdot 2}$	◀ Rules 2 and 3
	$A = 121p^8q^2$ square units	

Remember to label area problems with square units of measurement. Label volume problems with cubic units.

Example 4

Find the volume of a sphere with a radius of $2x^3y^2$ centimeters.

Plan Write the formula for the volume of a sphere.
Replace "r" with the given value.

Implement	Explain
$V = \frac{4}{3}\pi r^3$	◀ Volume of a sphere
	◀ Substitution
	◀ Rules 2 and 3
	◀ Commutative Property

Checkpoint

Find the volume of a rectangular prism with a length of $8x^2y$, a width of $3xy^3$, and a height of $3xy$ units.

Practice 1

Complete practice problems on a separate sheet of paper.

Simplify. Assume all variables are positive.

1) $(x^2y^3)^2 \cdot x^8$

2) $xy \cdot (xy^2)^3$

3) $(a^{12}b^6)^{\frac{1}{3}} \cdot a^2b^5$

4) $(xy)^4 \cdot (x^2y)^2$

5) $a^{\frac{1}{3}}b^{\frac{4}{3}} \cdot (ab^2)^{\frac{1}{3}}$

6) $(a^7b^3)^2 \cdot a^{-1}b^{-2}$

7) $2x \cdot (3x^5)^2$

8) $(3x^5y^7)^2 \cdot 5x$

9) $(5xy)^2 \cdot (2x^2y)^3$

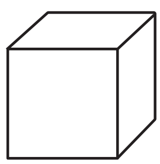
10) $x^{-1}y^{-5} \cdot (x^2y^3z^4)^5$

11) Find the value of n. $(5a)^2 \cdot 3a^nb^9 = 75a^7b^9$

For problems 12–17, choose the appropriate formula from the list below. Remember to label your answer with the proper units of measure.

Sphere	Rectangular prism	Rectangle	Square	Cylinder	Triangle	Cube
$V = \frac{4}{3}\pi r^3$	$V = lwh$	$A = lw$	$A = s^2$	$V = \pi r^2 h$	$A = \frac{1}{2}bh$	$V = s^3$

12) Find the volume of a cube with a side of $2x^3y$ feet.



$2x^3y$ feet

13) Find the area of a triangle with a base of $6x$ units and a height of $2x^2$ units.

14) Find the area of a rectangle with a length of $8a^3b^2$ centimeters and a width of $7a^5b^4$ centimeters.

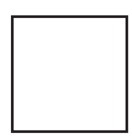
$7a^5b^4$ cm

$8a^3b^2$ cm

15) Find the volume of a cylinder in terms of pi when the height is $11x^3$ inches and the radius is x^5 inches.

16) Find the area of a rectangle with a width of ab^5 yards and a length of a^2c yards.

17) Find the area of a square with a side of $4x$ units.



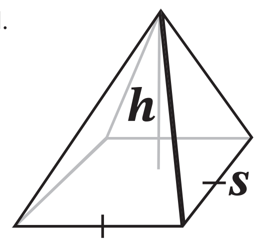
$4x$ units

Mastery Check

Show What You Know

The volume formula for a square pyramid is $V = \frac{1}{3}h \cdot s^2$, where h represents the height and s represents the length of the side of the base. The height of the pyramid is $3a^3$ inches. The length of a side of the base is $2ab^5$ inches.

A) Determine the volume of the pyramid.



B) Another student *incorrectly* determined the volume to be $4a^3b^{10}$ cubic inches. Using the provided work, determine the error and explain the misunderstanding.

Step 1 $V = \frac{1}{3}(3a^3) \cdot (2ab^5)^2$

Step 2 $V = \frac{3a^3}{3} \cdot 2^{1 \cdot 2} \cdot a^{1 \cdot 2} \cdot b^{5 \cdot 2}$

Step 3 $V = a \cdot 2^2 \cdot a^2 \cdot b^{10}$

Step 4 $V = 2^2 \cdot a^{1+2} \cdot b^{10}$

Step 5 $V = 4a^3b^{10}$ in^3

C) Evaluate your answer from part A when $a = 2$ and $b = 1$.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Practice 2

Complete practice problems on a separate sheet of paper.

Simplify. Assume all variables are positive.

1) $\left(x^{\frac{1}{3}}y^{6}\right)^{4} \cdot x^{\frac{1}{3}}y$

2) $\left(a^{3}b\right)^{\frac{1}{2}} \cdot a^{\frac{1}{2}}b$

3) $2^{3}ab \cdot \left(2a^{3}b\right)^{2}$

4) $\left(3a^{4}b^{7}\right)^{2} \cdot 3^{14}a^{6}b^{-2}$

5) $(xy)^{3} \cdot (xy)^{6}$

6) $x^{3}y \cdot \left(x^{5}y^{3}\right)^{3}$

7) $\left(3x^{2}y^{5}\right)^{3} \cdot 3xy$

8) $\frac{1}{2}x^{-8}y^{-1} \cdot \left(4x^{11}y\right)^{2}$

9) $(3ab)^{2} \cdot \left(3a^{2b}\right)$

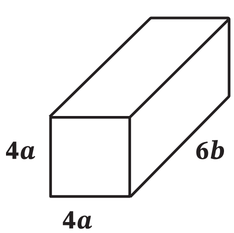
10) $\left(x^{8}y^{5}\right)^{2} \cdot \left(2x^{5}y\right)^{3}$

11) Find the value of n. $\left(a^{2}b^{3}\right)^{5} \cdot \left(a^{8}c\right)^{n} = a^{26}b^{15}c^{2}$

For problems 12–17, choose the appropriate formula from the list below. Remember to label your answer with the proper units of measure.

Sphere	Rectangular prism	Rectangle	Square	Cylinder	Triangle	Cube
$V = \frac{4}{3}\pi r^{3}$	$V = lwh$	$A = lw$	$A = s^{2}$	$V = \pi r^{2}h$	$A = \frac{1}{2}bh$	$V = s^{3}$

12) Find the volume of a rectangular prism with a length and width of $4a$ feet and a height of $6b$ feet.



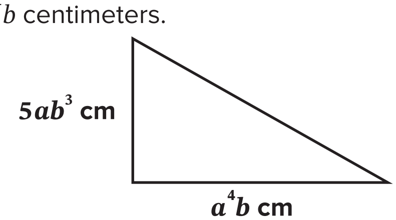
13) Find the volume of a sphere with a radius of $10x^{5}y^{3}$ units.

14) Find the area of a square with side lengths of $3ab$ inches.

15) Find the volume of a cube with sides of $5p^{7}q^{9}$ units.

16) Find the area of a rectangle with a length of $3x^{11}$ feet and a width of $13x^{5}y^{2}$ feet.

17) Find the area of a triangle with a height of $5ab^{8}$ centimeters and a base of $a^{4}b$ centimeters.



Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete practice problems on a separate sheet of paper.

1) Find the perimeter of a square with sides of $(x + 8)$ feet using the formula $P = 4s$.

For problems 2–4, use problem 1 to answer the questions.

2) What algebraic property is used to find the perimeter?

3) What would the perimeter be if $x = 4$?

4) Convert the perimeter from feet to yards.

5) Farmer McDonald is building a fence for their pigs. The total perimeter can be no more than 200 feet. The length must be at least 5 more than the width. Write a system of inequalities to represent fencing.

6) Write a system of equations. Do not solve. When the tens digit is subtracted from the units digit, the result is 5. The value of the digits is three times the sum of the digits.

7) What is the difference between an expression and an equation?

8) Write the equation. Do not solve. The variable x squared, plus twice x less six is zero.

For problems 9–12, classify the following as rational (Q) or irrational (I).

9) irrational + irrational =

10) rational · irrational =

11) rational + irrational =

12) rational + rational =

13) Solve the system of equations.
Name the method you used to solve.
$y = x + 1$
$y = 2x - 2$

14) Solve.
$6x - x + 3 = 4x + 7$

For problems 15–16, find the factor pairs of the following numbers.

15) 100

16) 48

Multiple Choice

_________ **17)** Evaluate the expression $-4ab^2 + a^2b$ when $a = 3$ and $b = 2$.

A) -400

B) -30

C) 30

D) 66

_________ **18)** Sammi purchased p pencils and b books for school. Sammi spent a total of \$26. The equation $p + 4b = 26$ represents the relationship between the number of books and pencils purchased. If the ordered pair $(6, 2)$ is the solution, what does this represent?

A) Sammi purchased 6 pencils and 2 books.

B) Sammi purchased 1 pencil and 4 books.

C) Sammi purchased 6 books and 2 pencils.

D) Sammi purchased 4 books and 26 pencils.