

LESSON 3

Negative Numbers, Multiplication

Multiplication is fast adding of the same number. In this case, it is fast adding of a negative number. The problem $(3) \times (-2)$ is a way of writing (-2) counted three times, or $(-2) + (-2) + (-2)$, or (-6) . Think of it as borrowing \$2 from someone for three days in a row. After three days you will owe \$6.

Example 1 $(-6)(+3) = (-18)$

Example 2 $(+7)(-6) = (-42)$

Once multiplying a negative number by a positive number clicks, consider what you would have if you were multiplying a negative number by a negative number. It will be the opposite of what we just learned, so we are back to being positive. There are only two options for a number: either it is negative or it is positive.

Since we first learned about multiplication, we always multiplied positive numbers by positive numbers. To understand a negative number times a negative number, let's review what we know so far with several more examples.

Example 3 $(+3)(+7) = (+21)$

Example 4 $(-3)(+7) = (-21)$

Example 5 $(+3)(-7) = (-21)$

The only option remaining is example 6.

Example 6 $(-3)(-7) = (+21)$

Think of negative anything as the opposite of what it was. We know that two wrongs don't make a right, but when multiplying two negative numbers, the product is a positive number. Here are a several more ways of thinking of this to help us understand a difficult concept.

In language, we know that a double negative is a positive. I used to ask students if they were going to the local town fair. They would reply that they weren't not going. I would respond by saying that I would see them there. In response to their puzzled expressions I would explain that if they were "Not, not going," then they were going.

Another way to think of it is using the idea of opposites as discussed in the previous lesson. Recall that $-(-21)$ is the same as $+21$. Using brackets for clarification, I can write $(-3)(-7)$ as $-[(3)(-7)]$ by moving the negative sign in front of the 3 outside of the brackets. After multiplying $(3)(-7)$, we have (-21) inside the brackets. Then putting it all together, we have $-[-21]$, which is $(+21)$.

Example 7 $(-12)(-5) = (+60)$

Have you observed the pattern that if you have two negative signs, you are positive? The same holds for four negative signs. Whenever you have an even number of negative signs the answer is positive, and an odd number of negative signs produces a negative answer. See figure 1.

Figure 1

$$\begin{aligned}(-12) &= (-12) \\ -(-12) &= (+12) \\ -[-(-12)] &= (-12) \\ -\{-[-(-12)]\} &= (+12)\end{aligned}$$

LESSON PRACTICE

3A

Multiply.

1. $(+5) \times (-6) =$

2. $(-6) \times (-7) =$

3. $(-9) \times (-10) =$

4. $(-10) \times (+12) =$

5. $(-5) \times (-8) =$

6. $(-16) \times (-11) =$

7. $(+4) \times (-15) =$

8. $(-18) \times (-6) =$

9. $(-16) \times (+12) =$

10. $(-17) \times (+3) =$

11. $(-18) \times (-4) =$

12. $(-24) \times (-5) =$

13. $(-11) \times (+16) =$

14. $(+3) \times (-24) =$

15. $(+8) \times (-12) =$

16. $(-10) \times (-16) =$

Write your answers as negative or positive numbers.

17. The team lost three games a week. What is its record at the end of six weeks?
18. Jim managed to lose 25 cents a day for 10 days. Express his loss as -25 cents a day. What was his total loss?
19. Karen's budget was short \$30 more every month. Express her shortfall as -30. How much was she short at the end of a year?
20. Peter's feet are 12 inches long. He stepped out the length and width of a room and found it was 10 feet by 12 feet. What is the area of the room?*

Note: Distance is expressed with a positive number. The area of a rectangle is found by multiplying the length times the width. The answer is always in square units.

LESSON PRACTICE

3B

Multiply.

1. $(+36) \times (-4) =$

2. $(-4) \times (-19) =$

3. $(-6) \times (-8) =$

4. $(-24) \times (-6) =$

5. $(-25) \times (-3) =$

6. $(-10) \times (+19) =$

7. $(-8) \times (+6) =$

8. $(-42) \times (+16) =$

9. $(-50) \times (-19) =$

10. $(+25) \times (-6) =$

11. $(+23) \times (-13) =$

12. $(-46) \times (-8) =$

13. $(-16) \times (-24) =$

14. $(-8) \times (-16) =$

15. $(-42) \times (-15) =$

16. $(-17) \times (+48) =$

Write your answers as negative or positive numbers.

17. I owed Sara three dollars. Express my debt as -3 . Because I forgot, she wants me to pay back two times as much. What is my debt?

18. The jar of face cream said it would take 10 years off the user's age with each application. If Ashley has used it five times, what is the effect on her age?

19. Tom's mortgage is \$682 a month. If he fails to pay for four months, what is the effect on his budget?

20. A pitcher gave up three runs in each inning (-3). What is the effect after nine innings?

LESSON PRACTICE

3C

Multiply.

1. $(+8) \times (-5) =$

2. $(-6) \times (+10) =$

3. $(-3) \times (-4) =$

4. $(-20) \times (+12) =$

5. $(+17) \times (+3) =$

6. $(-8) \times (-9) =$

7. $(-90) \times (+4) =$

8. $(+24) \times (-8) =$

9. $(+42) \times (-6) =$

10. $(-10) \times (-10) =$

11. $(+7) \times (-6) =$

12. $(-18) \times (-4) =$

13. $(-36) \times (+4) =$

14. $(+13) \times (-4) =$

15. $(-17) \times (-3) =$

16. $(+19) \times (-51) =$

Write your answers as negative or positive numbers.

17. Chris borrowed \$2 from me each day for five days. Express his debt for one day as a negative number, then multiply to find his total debt.
18. Mr. Brown loses 32 hairs every day. What is the result in 21 days?
19. The team lost four games a week. What is its record of losses at the end of 10 weeks?
20. Anna's garden is a rectangle that measures 7' by 14'. What is the area of her garden?

SYSTEMATIC REVIEW

3D

Multiply.

1. $(+17) \times (-6) =$

2. $(+22) \times (-11) =$

3. $(-5) \times (-9) =$

4. $(-10) \times (+5) =$

5. $(+6) \times (-7) =$

6. $(-16) \times (+9) =$

Change the signs as needed and solve.

7. $(+5) - (+10) =$

8. $(-6) + (-9) =$

9. $(+14) + (-3) =$

Find the fraction of the number.

10. $\frac{1}{2}$ of 20 =

11. $\frac{2}{3}$ of 15 =

12. $\frac{4}{9}$ of 27 =

Add or subtract. Leave answers in the form in which they occur.

13. $\frac{1}{10} + \frac{7}{10} =$

14. $\frac{5}{7} - \frac{1}{7} =$

15. $\frac{4}{8} + \frac{1}{8} =$

16. $\frac{7}{12} - \frac{3}{12} =$



QUICK REVIEW

When the numerator and denominator of a fraction are multiplied by the same number, the resulting fraction is “equivalent.” It has the same value as the original fraction, but is expressed in a different form.

EXAMPLE 1 $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

EXAMPLE 2 $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

You could continue to find as many equivalent fractions for $\frac{1}{2}$ as you wish. Fill in the missing numbers to make equivalent fractions.

17. $\frac{1}{3} = \frac{\quad}{6} = \frac{\quad}{9} = \frac{4}{\quad}$

18. $\frac{2}{5} = \frac{4}{\quad} = \frac{6}{15} = \frac{\quad}{20}$

Write your answers as positive or negative numbers.

19. The fuel tank leaks at a rate of two gallons a week. What is the effect on the contents after 13 weeks?
20. Matthew walked nine miles from the starting point. Then he turned around and walked two miles back. How far is he from his starting point?

SYSTEMATIC REVIEW

Multiply.

1. $(+16) \times (-10) =$

2. $(+17) \times (-10) =$

3. $(+23) \times (+11) =$

4. $(-8) \times (-4) =$

5. $(-7) \times (-8) =$

6. $(+10) \times (-11) =$

Change the signs as needed and solve.

7. $(+8) - (+19) =$

8. $(+17) + (-5) =$

9. $(-63) - (-50) =$

Find the fraction of the number.

10. $\frac{1}{3}$ of 18 =

11. $\frac{3}{7}$ of 49 =

12. $\frac{2}{11}$ of 44 =

Add or subtract. Leave answers in the form in which they occur.

13. $\frac{4}{5} - \frac{2}{5} =$

14. $\frac{5}{6} + \frac{1}{6} =$

15. $\frac{4}{13} + \frac{5}{13} =$

Fill in the missing numbers to make equivalent fractions.

16. $\frac{1}{4} = \frac{\quad}{8} = \frac{3}{\quad} = \frac{\quad}{16}$

17. $\frac{5}{8} = \frac{\quad}{\quad} = \frac{15}{24} = \frac{\quad}{\quad}$

18. By working very hard, George painted $\frac{1}{8}$ of the house on Monday and $\frac{2}{8}$ on Tuesday. What part of the house has been painted?

Write your answers as positive or negative numbers.

19. Bill ordered a book that cost \$25. By mistake he sent the company \$30. They sent back an invoice that showed his account balance as a negative number. What was the number?
20. A stunt pilot flew around the perimeter of our town. If the town is a square that measures five miles on each side, what is the area of the town? (A square is a special kind of rectangle.)

SYSTEMATIC REVIEW

3F

Multiply.

1. $(+14) \times (-5) =$

2. $(-18) \times (+11) =$

3. $(-9) \times (-12) =$

4. $(+14) \times (-6) =$

5. $(-19) \times (-23) =$

6. $(-19) \times (+17) =$

Change the signs as needed and solve.

7. $(+32) + (-18) =$

8. $(-94) + (-7) =$

9. $(+58) - (+100) =$

Find the fraction of the number.

10. $\frac{1}{5}$ of 20 =

11. $\frac{2}{3}$ of 21 =

12. $\frac{3}{10}$ of 50 =

Add or subtract. Leave answers in the form in which they occur.

13. $\frac{2}{3} - \frac{1}{3} =$

14. $\frac{4}{7} - \frac{2}{7} =$

15. $\frac{1}{9} + \frac{5}{9} =$

Fill in the missing numbers to make equivalent fractions.

16. $\frac{1}{6} = \frac{\quad}{\quad} = \frac{\quad}{18} = \frac{4}{\quad}$

17. $\frac{3}{7} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{12}{28}$

18. Five-twelfths of the pizza was left over. Austin then ate three-twelfths of a whole pizza. How much pizza was left when Austin was finished?

Write your answers as positive or negative numbers.

19. Kelly's uncle sent her \$15 a month. What was the effect on her income in four months?
20. Thinking her uncle was going to send her \$20 a month, Kelly promised that amount to her sister. What is the combined effect of #19 and #20 on Kelly's budget during that four months?

Apply the math skills you already have to solve these problems.

1. If a square has a perimeter of 68, what is its area?

2. A rectangle has a length of eight and a width of six. What is the area of the rectangle? If the length and width of the rectangle are both doubled, what is the new area?

How many times the original area is the new area?

3. Find half of the original length and width of the rectangle in #2, then find the new area. What part of the original area is the new area?

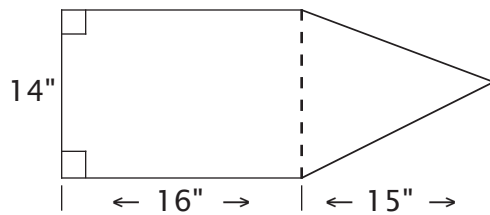
4. If the length and width of a rectangle are both tripled, what will be the effect on the area of the rectangle? Sketch and label two rectangles to illustrate your answer.

5. What is 38.98 rounded to the nearest tenth?

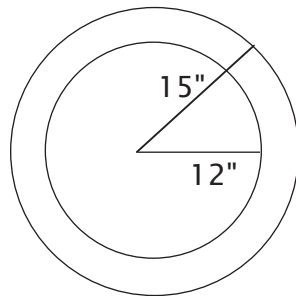
6. Several people are standing in line. Starting at one end, Tony is the third person. Starting at the other end, he is the eleventh person. How many people are in line?

If you know how to find the area of simple geometric shapes, you can combine these skills to find the area of more complex shapes. If you need to review area formulas, check the “Symbols and Tables” pages at the back of this book.

7. What is the area of the figure shown? Dotted lines have been drawn to show how it is made up of two different geometric shapes.



8. The figure below represents a circular piece of metal whose center has been cut out to leave a ring. What is the area of the remaining ring? The straight lines represent the radii of the two circles.



TEST

Multiply.

1. $(-20) \times (-4) =$

2. $(+19) \times (-3) =$

3. $(-30) \times (-17) =$

4. $(-27) \times (+8) =$

5. $(-9) \times (+2) =$

6. $(-7) \times (-29) =$

Change the signs as needed and solve.

7. $(+33) - (-46) =$

8. $(-27) + (-10) =$

9. $(-41) - (-20) =$

Find the fraction of the number.

10. $\frac{1}{3}$ of 24 =

11. $\frac{2}{5}$ of 15 =

12. $\frac{3}{7}$ of 28 =

Add or subtract. Leave answers in the form in which they occur.

13. $\frac{5}{8} - \frac{3}{8} =$

14. $\frac{7}{10} - \frac{1}{10} =$

15. $\frac{1}{4} + \frac{1}{4} =$

Fill in the missing numbers to make equivalent fractions.

16. $\frac{1}{5} = \frac{\quad}{\quad} = \frac{\quad}{15} = \frac{4}{\quad}$

17. $\frac{2}{3} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{8}{12}$

18. Emily did one-fifth of the chores and Madison did three-fifths of them. What part of the chores has been done?

Write your answers as positive or negative numbers.

19. Elizabeth spent \$4 a day on lunch for five days. Write the daily cost of the lunch as a negative number and multiply to find the total change in the amount of money she has.
20. During the drought, the water level in the lake fell two feet (-2) every week. What was the effect on the water level in six weeks?

4. $(-6) - (+3) =$
 $(-6) + (-3) = -9$
5. $(-13) - (-14) =$
 $(-13) + (+14) = +1$
6. $(+39) - (-8) =$
 $(+39) + (+8) = +47$
7. $(+76) - (+26) =$
 $(+76) + (-26) = +50$
8. $(-24) - (+85) =$
 $(-24) + (-85) = -109$
9. $(-35) + (-42) = -77$
10. $(+50) + (-51) = -1$
11. $(+62) - (-12) =$
 $(+62) + (+12) = +74$
12. $(-23) - (-8) =$
 $(-23) + (+8) = -15$
13. $\frac{3}{10} + \frac{3}{10} = \frac{6}{10}$
14. $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$
15. $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$
16. $\frac{9}{11} - \frac{6}{11} = \frac{3}{11}$
17. $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ of the candies
18. $35 \div 5 = 7$
 $7 \times 3 = 21$ birds
19. $(-4) + (-5) = -9$ hours
20. 9 hours assigned - 10
hours worked = -1 hours
left to work
5. $(+12) - (-48) =$
 $(+12) + (+48) = +60$
6. $(-8) - (+5) =$
 $(-8) + (-5) = -13$
7. $(-13) + (-11) = -24$
8. $(-8) - (+25) =$
 $(-8) + (-25) = -33$
9. $(+37) - (-40) =$
 $(+37) + (+40) = +77$
10. $(-51) + (+73) = +22$
11. $(-62) + (-65) = -127$
12. $(-16) - (-18) =$
 $(-16) + (+18) = +2$
13. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$
14. $\frac{6}{7} - \frac{5}{7} = \frac{1}{7}$
15. $\frac{5}{9} - \frac{1}{9} = \frac{4}{9}$
16. $\frac{7}{20} + \frac{3}{20} = \frac{10}{20}$
17. $30 \div 3 = 10$
 $10 \times 2 = 20$ right
 $30 - 20 = 10$ wrong
18. $\frac{4}{10} - \frac{3}{10} = \frac{1}{10}$ of a pie
19. $(\$ - 100) + (\$ + 60) = \$ - 40$
20. -7 brownies

Systematic Review 2F

1. $(+4) - (+10) =$
 $(+4) + (-10) = -6$
2. $(-3) - (-6) =$
 $(-3) + (+6) = +3$
3. $(-2) - (+6) =$
 $(-2) + (-6) = -8$
4. $(+7) - (-14) =$
 $(+7) + (+14) = +21$

Lesson Practice 3A

1. $(+5) \times (-6) = -30$
2. $(-6) \times (-7) = +42$
3. $(-9) \times (-10) = +90$
4. $(-10) \times (+12) = -120$
5. $(-5) \times (-8) = +40$
6. $(-16) \times (-11) = +176$
7. $(+4) \times (-15) = -60$
8. $(-18) \times (-6) = +108$
9. $(-16) \times (+12) = -192$
10. $(-17) \times (+3) = -51$
11. $(-18) \times (-4) = +72$
12. $(-24) \times (-5) = +120$

13. $(-11) \times (+16) = -176$
14. $(+3) \times (-24) = -72$
15. $(+8) \times (-12) = -96$
16. $(-10) \times (-16) = +160$
17. $(-3) \times (+6) = -18$ games
18. $(\$ - .25) \times (+10) = \$ - 2.50$
19. $(\$ - 30) \times (+12) = \$ - 360$
20. $(+10) \times (+12) = +120 \text{ ft}^2$

Lesson Practice 3B

1. $(+36) \times (-4) = -144$
2. $(-4) \times (-19) = +76$
3. $(-6) \times (-8) = +48$
4. $(-24) \times (-6) = +144$
5. $(-25) \times (-3) = +75$
6. $(-10) \times (+19) = -190$
7. $(-8) \times (+6) = -48$
8. $(-42) \times (+16) = -672$
9. $(-50) \times (-19) = +950$
10. $(+25) \times (-6) = -150$
11. $(+23) \times (-13) = -299$
12. $(-46) \times (-8) = +368$
13. $(-16) \times (-24) = +384$
14. $(-8) \times (-16) = +128$
15. $(-42) \times (-15) = +630$
16. $(-17) \times (+48) = -816$
17. $(\$ - 3) \times (+2) = \$ - 6$
18. $(-10) \times (+5) = -50$ years
19. $(\$ - 682) \times (+4) = \$ - 2,728$
20. $(-3) \times (+9) = -27$ runs

Lesson Practice 3C

1. $(+8) \times (-5) = -40$
2. $(-6) \times (+10) = -60$
3. $(-3) \times (-4) = +12$
4. $(-20) \times (+12) = -240$
5. $(+17) \times (+3) = +51$

6. $(-8) \times (-9) = +72$
7. $(-90) \times (+4) = -360$
8. $(+24) \times (-8) = -192$
9. $(+42) \times (-6) = -252$
10. $(-10) \times (-10) = +100$
11. $(+7) \times (-6) = -42$
12. $(-18) \times (-4) = +72$
13. $(-36) \times (+4) = -144$
14. $(+13) \times (-4) = -52$
15. $(-17) \times (-3) = +51$
16. $(+19) \times (-51) = -969$
17. $(\$ - 2) \times (+5) = \$ - 10$
18. $(-32) \times (+21) = -672$ hairs
19. $(-4) \times (+10) = -40$ losses
20. $(+7) \times (+14) = +98 \text{ ft}^2$

Systematic Review 3D

1. $(+17) \times (-6) = -102$
2. $(+22) \times (-11) = -242$
3. $(-5) \times (-9) = +45$
4. $(-10) \times (+5) = -50$
5. $(+6) \times (-7) = -42$
6. $(-16) \times (+9) = -144$
7. $(+5) - (+10) =$
 $(+5) + (-10) = -5$
8. $(-6) + (-9) = -15$
9. $(+14) + (-3) = +11$
10. $20 \div 2 = 10$
 $10 \times 1 = 10$
11. $15 \div 3 = 5$
 $5 \times 2 = 10$
12. $27 \div 9 = 3$
 $3 \times 4 = 12$
13. $\frac{1}{10} + \frac{7}{10} = \frac{8}{10}$
14. $\frac{5}{7} - \frac{1}{7} = \frac{4}{7}$
15. $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$
16. $\frac{7}{12} - \frac{3}{12} = \frac{4}{12}$