Calculus Placement Instructions

This assessment will help determine if this level of Math-U-See is a good place for your child to start. Each level of Math-U-See builds upon the concepts taught in previous levels. Successful placement involves finding the highest level your child has fully mastered and placing them one level above that.

1 Prior to beginning the assessment:

- Understand that the goal isn't to get all the questions correct. We are determining which concepts they have not yet mastered.
- Encourage your child and let them know that this is an assessment and NOT a test.
- Recognize they might already know some of the concepts taught in this level.
- Let your child know there may be questions they don't yet understand.
- Print the assessment and ensure you have a pencil and eraser.
- Your child may want extra paper to work through the questions.

2 Let your child know while taking the assessment:

- If they don't understand or can't do a question have them move to the next one.
- If they want to attempt a question but are not sure they understand it, have them mark it with a happy face.
- If they cannot answer 3 or more questions in a row, it is okay to stop doing this assessment.

3 Grading the assessment:

- A question that your child has marked with a happy face indicates to you that this concept is not completely understood and must be reviewed.
- For incorrect answers, ask your child how they arrived at their answer. If they understand the concept, they should be able to correct the mistake on their own. This is considered a computational error. For the sake of this assessment do not mark this as incorrect.
- If there are only one or two concepts they need to learn or review from a given level, it may be possible to just remediate those and start in the next level higher.

4 Analyzing the results:

Most answers are incorrect or have happy faces.	5 or more answers are incorrect or have happy faces.	Most answers are correct and there are no happy faces.
Have them try the assessement for **Pre-Calculus**	**Your child is ready for** **Calculus**	**Your child should review any concepts they find challenging**

If you have questions after your child has taken the assessment, please contact us with the results and we will be able to help you determine the best level for them.

FINAL EXAM (100 points possible, 5 points a question)

I. Let $f(x) = x^2$, $g(x) = 3^x$ and $H(x) = 1 - x^2$.

1. Find $f(g(2))$.

2. Find $f(x) + 2H(x)$.

3. Find $\lim_{x\to 0} g(x)$.

4. Find $\lim_{x\to\infty} \frac{H(x)}{f(x)}$.

5. Using the definition, find the first derivative of $f(x) = 2x^2$.

II. Find the first derivative for each of the following.

1. $r = \sqrt{1-2\theta}$; find $\frac{dr}{d\theta}$.

2. $w = xe^{2x}$; find $\frac{dw}{dx}$.

3. $y = \ln(\sin(\theta))$; find $\frac{dy}{d\theta}$.

III. Integrate the following:

1. $\int_1^{e^2} \frac{\sqrt{\ln(x)}}{x}\,dx =$

2. $\int \sin(2x)\cos(2x)\,dx =$

IV. Find all extrema and any inflection points for $f(x) = -2x^3 + 6x^2 - 3$. Find the regions of concavity. Include a sketch.

V. Find the area of the region bounded by the curve $y = \sqrt{x - 1}$ and the lines $y = 1$ and $y = 2$.

VI. Evaluatethefollowing limits.

1. $\lim_{x \to 3} \frac{x^2 - 9}{x^2 + x} =$

2. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} =$

3. $\lim_{x \to 0} \frac{\cos(x) - 1}{\sin(x)} =$

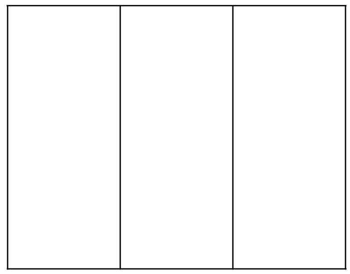
4. $\lim_{x \to \infty} \left(1 + \frac{5}{x}\right)^{2x} =$

5. $\lim_{x \to \infty} \frac{3x^2 - 4x + 7}{x^2 + 3} =$

VII. Find the equation of the line tangent to $x^2 - \ln(x)$ at (1, 1).

VIII. Find the area between $y = x^2 + 1$ and $y = x + 3$. Include a sketch.

IX. Sixteen meters of fencing are to be used to make animal pens. Each of the three pens will be equal in size. Find the dimensions of the total area, which would be a maximum.



Calculus Pre/Post Placement Test
Answer Key

I.

1. $f(g(x)) = f(3^x) = (3^x)^2 = 3^{2x}$

   $f(g(2)) = 3^{2(2)} = 3^4 = 81$

2. $f(x) + 2H(x) = x^2 + 2(1 - x^2)$

   $= x^2 + 2 - 2x^2$

   $= 2 - x^2$

3. $\lim_{x\to 0} 3^x = 3^0 = 1$

4. $\lim_{x\to\infty} \frac{H(x)}{f(x)} \lim_{x\to\infty} \frac{1-x^2}{x^2} = \lim_{x\to\infty} \frac{\frac{1}{x^2} - 1}{1} = -1$

5. $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

   $= \lim_{h\to 0} \frac{2(x+h)^2 - 2x^2}{h}$

   $= \lim_{h\to 0} \frac{(2x^2 + 2xh + h^2) - 2x^2}{h}$

   $= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$

   $= \lim_{h\to 0} \frac{4xh + 2h^2}{h}$

   $= \lim_{h\to 0} (4x + 2h) = 4x$

II.

1. $r = \sqrt{1 - 2\theta}$ $\quad \frac{dr}{d\theta} = \frac{1}{2}(1 - 2\theta)^{-\frac{1}{2}}(-2)$

   $\frac{dr}{d\theta} = \frac{-1}{\sqrt{1 - 2\theta}}$

2. $w = xe^{2x}$ $\quad \frac{dw}{dx} = x(2e^{2x}) + e^{2x}(1)$

   $= e^{2x}(2x + 1)$

3. $y = \ln(\sin(\theta))$ $\quad \frac{dy}{d\theta} = \frac{1}{\sin(\theta)} \cdot \cos(\theta)$

   $= \cot(\theta)$

III.

1. Let $u = \ln(x)$; $du = \frac{1}{x} dx$

   $\int_1^{e^2} \frac{\sqrt{\ln(x)}}{x} dx = \int_1^{e^2} u^{\frac{1}{2}} du$

   $= \left. \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right|_1^{e^2}$

   $= \left. \frac{2}{3}(\ln(x))^{\frac{3}{2}} \right|_1^{e^2}$

   $= \frac{2}{3}\left[(\ln(e^2))^{\frac{3}{2}} - (\ln(1))^{\frac{3}{2}}\right]$

   $= \frac{2}{3}(\sqrt{8})$

   $= \frac{2}{3}(2\sqrt{2}) = \frac{4\sqrt{2}}{3}$

2. Let $u = \sin(2x)$; $du = \cos(2x) \cdot dx \cdot 2$

   $\frac{1}{2}\int \sin(2x)\cos(2x)dx \cdot 2$

   $= \frac{1}{2}\int u du$

   $= \frac{1}{2}\frac{u^2}{2} + C$

   $= \frac{1}{4}(\sin^2(2x)) + C$

IV. $f(x) = -2x^3 + 6x^2 - 3$

$f'(x) = -6x^2 + 12x$

$-6x(x - 2) = 0$

$x = 0$, $x = 2$ are critical points.

$f''(x) = -12x + 12$

$-12(x - 1) = 0$

$x = 1$ possible inflection point

$f''(0) = 12$ so $x = 0$ is a min.

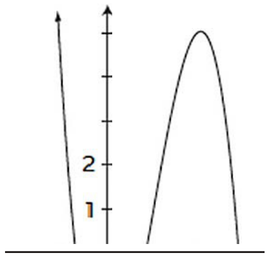
$f''(2) = -12$ so $x = 2$ is a max.

f''

\+

x=1is an inflection point

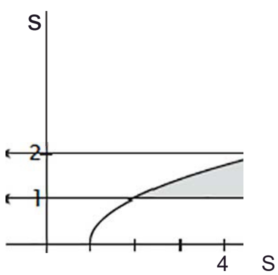
x	y
0	- 3
1	1
2 3	5 3 -
- 1	5



/is concave up from (-«>, 1)
and concave down from (1, oo).

V. y= ©

v 2 = x - l ; x = v 2 + 1



r (v2+1)dy= (+ + y)

= (: + 2) - (+ +1) = 10 3

VI.

1. $\lim_{x\to 3} \frac{x^2-9}{x^2+x} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x(x+1)} = 0$

2. $\lim_{X\to 3+} \frac{x^2-9}{X-3} = \lim_{X\to 3} \frac{(x+ \text{TM}}{X} = 6$

3. $\lim_{x\to 0} \frac{\cos(x)-1}{\sin(x)} \quad \lim_{x\to 0} \frac{-\sin(x)}{\cos(x)}$ 0 using LR

4. $\lim_{x\to\infty} \frac{3x^2-4x+7}{x^2+3} = \lim_{x\to\infty} \frac{3-\frac{4}{x}+\frac{7}{x^2}}{1+\frac{3}{x^2}} = 3$

VIL $f(x)=x^2-\ln(x)$

$f'(x) = 2x - \frac{1}{x}$

$f'(1) = 1$

y= m x + b

1= 1(1)+ b

b = O

So the equation of the tangent line is Y=X.

VIII. $y=x^2+1$; $y= x+3$

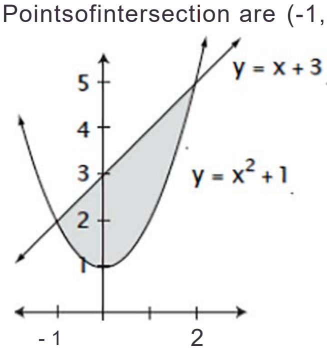
$x^2+1=x+3$

$x^2-x-2=0$

$(x+1)(x-2)=0$

$x=-1, 2$

Pointsofintersection are (-1, 2) and (2, s).



$$\int_{-1}^{2}[(x+3)-(x+1)]dx$$

$$=\int_{-1}^{2}(-x^2+x+2)dx$$

$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Big|_{-1}^{2}$$

$$=\left(-\frac{8}{3}+2+4\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)$$

$$=-\frac{8}{3}-\frac{5}{6}+8=4\frac{1}{2}$$

IX. $2x+4y=16$

$$y=4-\frac{1}{2}x$$

$$A(x)=xy$$

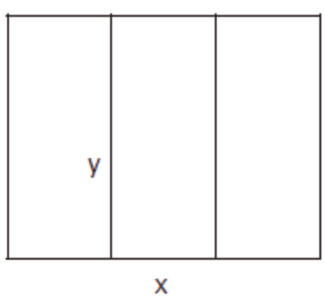
$$=x\left(4-\frac{1}{2}x\right)$$

$$=4x-\frac{1}{2}x^2$$

$A'(x)=4-x=0$ when $x=4$

$A''(x)=-1$

so $A''(4)=-1$ which yields a max.



If $x = 4$, when $y=4-\frac{1}{2}(4)=2$.

The dimensions of the largest possible area would be 4 ft by 2 ft.